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Soil Moisture Accounting Based Sediment Yield Modelling of Arid Drainage Basin of Rajasthan, India

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ABSTRACT

This study evaluates the adequacy of soil moisture accounting based sediment graph models (SMA-SGMs) for an arid upland drainage basin of 1520 km^2, those are originally evaluated for humid/sub-humid watersheds of size as large as 28 km^2. These SGMs fit realistic sediment graphs (SGs) with Nash-Sutcliffe efficiency (*NSE*) of 92.18 to 95.66%, besides conserving their mass closely. These SGMs are physically more plausible and precise than the conventional regression relations. Thus, proposed models proves their efficacy over arid Jasnagar sub-basin of Luni river, Rajasthan (India) by simulating and validating sediment response with negligible relative error (*RE*) in total sediment load/volume (Q_S), peak sediment load (Q_{PS}) and time to peak sediment load (t_{PS}) viz. $RE(Q_S)$=7.24%, $RE(Q_{PS})$=0.43% and $RE(t_{PS})$ =0, respectively.

Key words : *Peak sediment outflow, Sediment graph model, Soil moisture, Time to peak sediment outflow, Total sediment outflow.*

Introduction

Rajasthan shares 50.64% of Indian arid zone (31.7 mha). Which is characterized by perpetual climatic stresses-erratic meagre rainfall; extreme temperatures; high evaporation and soil erosion. Erosion devastatingly impacts soil fertility, water supply, flood control, groundwater, irrigation, hydropower, recreation, fishing, tourism, etc. As pesticides residues; absorbed nutrients, organic compounds; pathogens and viruses; heavy/radioactive metals conveyed by sediments affects the water purity, transparency and quality and eventually lead to eutrophication. Studies of pollutants and hydropower operations require peak rather than average or total sediment rates. Thus, precise estimates of sediment graph has a high priority in soil and water conservation, reservoir operation, river morphological studies, agricultural project planning, flood frequency analysis, hydraulic and sanitary designs, and water-quality modelling.

Huge sediment loads in arid regions (Sharma *et al.*, 1992) are attributed to flash floods (Reid and Frostick, 1987), excessive weathering (Goudie and Wilkinson, 1977) of erodible aeolian deposits (Jones, 1981) due to sparse vegetation cover (Pilgrim *et al.*, 1988) and intensive anthropogenic activities. The sediment response of arid region is far less detailed due to spatial heterogeneities, complexity of hydrogeological processes and infrequent runoff events (Jones, 1981; Reid and Frostick, 1987). The paucity of data due to sparse gauging network; requirement of huge funds, substantial land area, skilled field personnel; equipment failure and inaccessibility often

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make repeated field studies unfeasible (Pilgrim *et al.*, 1988).

Universally accepted sediment yield formulae are still lacking (Kothyari *et al.*, 1996, 2002) in spite of several empirical, conceptual and process-based models. The empirical models e.g. universal soil loss equation (USLE) are condition specific. Some of the popular conceptual models are: unit SG (USG) (Rendon-Herrero, 1978) and instantaneous USG (IUSG) (Williams, 1978). Johnson (1943) derived suspended matter distribution graphs. Bruce *et al.* (1975) developed a SGM based on erosion and transport capacity. Nash's (1957) IUSG model was skilfully employed by Singh *et al.* (1982), Kumar and Rastogi (1987), Sharma *et al.* (1992), Raghuwanshi *et al.* (1994), Sharma and Murthy (1996), Gracia Sánchez (1996), Lee and Singh (1999, 2005), Rai and Mathur (2007), Singh *et al.* (2008), and Bhunya *et al.* (2010). The regression relationships used in USG- or IUSG-based models are not much reasonable (Raghuwanshi *et al.*, 1994). Furthermore, these models do not account for major runoff and, in turn, sediment yielding watershed characteristics. The USLE and Soil Conservation Service-Curve Number (SCS-CN) based models (Mishra *et al.*, 2006a,b) accounting for initial abstraction (*Ia*) and initial soil moisture (V_0), are unsuitable for time distributed sediment modelling. Singh *et al.* (2008) and Bhunya *et al.* (2010) SGMs couples popular Nash's IUSG, SCS-CN method and power law. Tyagi *et al.* (2008) developed a time distributed sediment yield model (SYM) utilizing the SCS-CN based infiltration model. More recently, Gupta *et al.* (2019) developed four SGMs based on Nash's IUSG, SCS-CN method and Power law coupled with SMA procedure assuming that if a soil is saturated, rainfall becomes runoff (Michel *et al.*, 2005) adopted in present study to predict SGs.

Theoretical Considerations

Using Nash's (1957) IUH, the sediment outflow from last (n_s^{th}) reservoir is (Singh *et al.*, 2008):

$$Q_{sn_s}(t) = \frac{1}{K_s \Gamma(n_s)} \left(\frac{t}{K_s}\right)^{n_s-1} e^{\frac{-t}{K_s}}, K_s > 0, t > 0 \qquad ..(1)$$

Γ() is gamma function; n_s is no. of linear reservoirs; K_s is storage coefficient (hr); *t* is time since beginning of outflow (hr). Assuming at $t = t_{ps}$ -time to peak sediment rate, $dQ_{sn}(t)/dt = 0$ and differentiating Eq. (1) w.r.t. (*t*):

$$K_S = t_{PS}/(n_s - 1) \qquad ..(2)$$

Combining Eq. (1) & (2), we get IUSG ordinates Q_{sns} (t) at time *t* in hr^{-1} (Singh *et al.*, 2008). Convolution of the IUSG and mobilized sediment gives a SG. Bhunya *et al.* (2003) proposed reliable expressions to derive shape parameter (n_s) from known q_{ps} (1/hr) and t_{ps} (hr):

$n_s = 5.53\beta_s^{1.75} + 1.04$ IF($0.01 < \beta_s < 0.35$) & $n_s = 6.29\beta_s^{1.998} + 1.157$ IF ($\beta_s > 0.35$) .. (3a-b)

Where, $\beta_s = q_{ps} \times t_{ps}$ and $q_{ps} = Q_{PS}/Q_S$ [(kg/s)/(hr).(kg/s)]. Using above IUSG concept, Gupta *et al.* (2019) developed four SGMs denoted as SMA-SGM$_{1-4}$ (Table 1).

Study Area

The Luni river and its tributaries draining sub-basins of 104 to total 34,866 km^2 from Aravalli hills to the Rann of Kachchh and form the only integrated drainage system in NW arid India. Three observed storm SGs were obtained from Sharma (1993). Herein, four SMA-SGM$_{1-4}$ were applied/calibrated and validated over (02+01) storm events of Jasnagar sub-basin (1520 km^2) of Luni river, to test their workability for arid basin. The SCS-CN=40.53 is derived from the study of National Institute of Hydrology (NIH), Roorkee (NIH, 1997-98).

Performance Evaluation

The SMA-SGMs is evaluated both graphically and statistically. For '*n*' no. of SG ordinates,

$$\frac{Nash-Sutcliffe}{Efficiency} = \left[1 - \frac{\sum_{i=1}^{n}\left[i^{th}\text{obs. SG ordinate} - i^{th}\text{comp. SG ordinate}\right]^2}{\sum_{i=1}^{n}\left[i^{th}\text{obs. SG ordinate} - \overline{\text{mean of all obs. SG ordinates}}\right]^2}\right] \times 100 \quad (4)$$

NSE ranges between ∞ and 1 (1 inclusive), $NSE = 1$ exhibits optimal/perfect fit.

$$Absolute\ RE = \left|\frac{Observed\ Value - Computed\ Value}{Observed\ Value}\right| \times 100 \quad ..(5)$$

$R(Q_S)$, $RE(Q_{PS})$, $RE(t_{PS})$ = RE's in total-, peak-, and time to peak-sediment outflow, respectively. Higher RE reflects poorer performance & vice-versa. RE=0 exhibits a perfect fit.

Results and Discussion

An arid basin sediment response is executed and validated using SMA-SGMs as illustrated in Figs. (1-3). Table 2 shows the calibration and validation parameters and observed/modelled SGs properties evaluated using absolute *RE* and *NSE*. Model parameters (α, β, κ, θ, λ and *A*) were optimized using

GRG-nonlinear programming algorithm, employing minimization of $RE(Q_{PS})$ as an objective function. α varies model-wise from 0.15 (SMA-SGM_{2-4}) to 0.49 (SMA-SGM_1) in calibration. Whereas, (*α*, *k*, θ and λ) attain consistent values (0.10, 0.04, 0.04 and 0.04) during calibration. *k* is the Horton's decay constant, ratio of uniform rainfall intensity (i_0) to the potential maximum retention (*S*), which is affected by rainfall intensity, soil type, land use, and hydrologic conditions (Mein and Larson, 1971). The watershed specific θ (=V_0/S) usually varies between 0 to 1 (Michel *et al.*, 2005) as observed. Model response to diverse geologic and climatic settings is governed by λ and its calibrated value (=0.04) is close to the recommended value ≤ 0.05 (Woodward *et al.*, 2003). The watershed specific parameter *S* (= 25400/CN-254) bears a constant value (= 372.70 mm) during calibration and validation. The event specific parameter, *A* is optimized so as to match sediment delivery ratio, SDR=*Y*/*A* with SDR = 0.51 × $A_W^{-0.11}$ (SCS, 1972) and it ranges from 3.78 to 5.68 in calibration. These SDR expression were used again to estimate event specific *A* for validation. Such estimation of *S* and *A* diminishes the effect of conventional direct optimization on model complexity. As uniqueness/consistency of parameters has been preserved enhancing physical interpretability resulting in goodness of fit of SGMs, which was lacking in earlier studies. The closer agreement of SGs during calibration as well as validation is justified by their properties (Q_{PS}, t_{PS} and Q_S) (Table 2). The SMA-SGM_{1-4} performed analogously during calibration as well as validation as shown by the identical *RE* and *NSE* (Table 2) of the derived SGs. Hence, for graphical evaluation with observed SGs, the SMA-SGM_1 based SG is employed as representative of modelled SGs as shown in Figs. 1-3.

The model wise average calibrated parameters (α, β, *k*, θ, λ) were (Table 2) used for validating the SMA-SGMs over the subsequent storm event of Jasnagar basin. The average value of α vary from 0.15 (SMA-SGM_{2-4}) to 0.48 (SMA-SGM_1). The average values of β, κ, θ, and λ (≈ 0.10, 0.04, 0.04 and 0.04, respectively) used for validation are consistent. The event specific *A=Y/SDR* derived for validation is 52.79. As the event wise as well as model wise inconsistency of parameters restricts their employment in validation as well as for estimating sediment response of geomorphologically similar ungauged basins. The event wise NSE values varies from 92.18% (Validation) to 95.66% (Calibration)

Table 1. The SMA-SGM_{1-4} proposed by Gupta *et al.* (2019)

Model [Hydrologic Condition]	**SMA-SGM1 [Initial Soil Moisture, $V_0 = 0$ and Initial Abstraction, $I_a = 0$]**	**SMA-SGM2 [$V_0 \neq 0$ and $I_a = 0$]**
Parameters	α, β, k, A, and n_s	$\alpha, \beta, k, \theta, S, A$, and n_s
Sediment Graph (kg/s)/hr	$Q_S(t)=\dfrac{\alpha AA_w[(kt/(kt+1))^2]^{\beta}(n_s-1)^{n_s}}{[t_{ps}\Gamma(n_s)][(t/t_{ps})e^{-(t/t_{ps})}]^{(n_s-1)}}$	$Q_S(t)=\dfrac{\alpha AA_W[[\theta(1+kt)+kst/(1+kt)(1+\theta)]^2]^{\beta}(n_s-1)^{n_s}}{[t_{ps}\Gamma(n_s)][(t/t_{ps})e^{-(t/t_{ps})}]^{(n_s-1)}}$
Model [Hydrologic Condition]	**SMA-SGM3 [$V_0 = 0$ and $I_a \neq 0$]**	**SMA-SGM4 [$V_0 \neq 0$ and $I_a \neq 0$]**
Parameters	$\alpha, \beta, k, \lambda, S, A$, and n_s	$\alpha, \beta, k, \theta, \lambda, S, A$, and n_s
Sediment Graph (kg/s)/hr	$Q_S(t)=\dfrac{\alpha AA_W[[kst/(1+kt)(1+\lambda)]^2]^{\beta}(n_s-1)^{n_s}}{[t_{ps}\Gamma(n_s)][(t/t_{ps})e^{-(t/t_{ps})}]^{(n_s-1)}}$	$Q_S(t)=\dfrac{\alpha AA_W[[\theta(1+kt)+kst/(1+kt)(1+\theta+\lambda)]^2]^{\beta}(n_s-1)^{n_s}}{[t_{ps}\Gamma(n_s)][(t/t_{ps})e^{-(t/t_{ps})}]^{(n_s-1)}}$

A_w is the watershed area (km^2), *A* is the potential maximum erosion ((kg/s)/km^2)

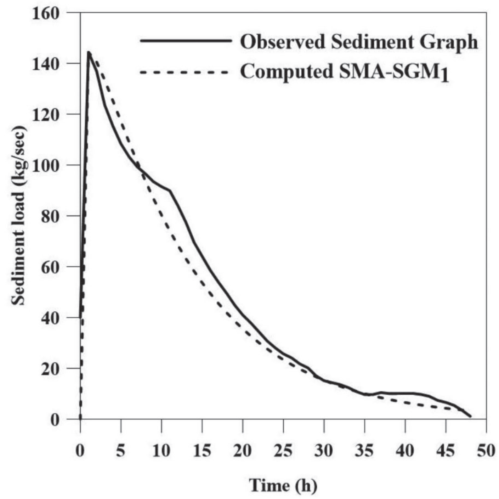

Fig. 1. Observed *vs.* Computed Sediment Graph of Calibration Event (10-11/07/1981)

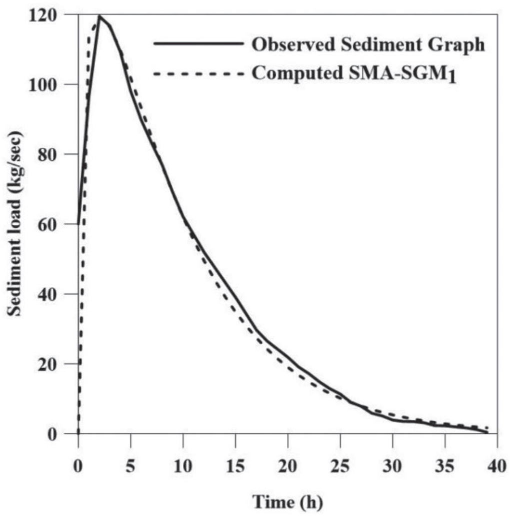

Fig. 2. Observed *vs.* Computed Sediment Graph of Calibration Event (24-26/07/1982)

(Table 2). The resulting $RE(Q_{PS})$ and $RE(Q_s)$ are ranging from 0 to 0.43% and 0.78 to 7.24%, respectively. Notably, t_{PS} matched exactly in both calibration and validation as, $RE(t_{PS})$=0. The existing conceptual SGMs/SYMs have however performed with enhanced errors in (Q_s, Q_{PS}, t_{PS}) and some of them are still lacking in validation. For an e.g. Kothyari *et al.*, (1996) [RE(Q_{PS}) = 56.63% and RE(Q_s) =89.06%], Kothyari *et al.* (2002) [91.48% and 87.74%], Rai and Mathur (2007) [61.5% and 37.8%], Tyagi *et al.* (2008) [24.86% and 42.15%], Singh *et al.* (2008) [12.95% and 15.75%], Bhunya *et al.* (2010) [16.56% and 10.04%], Gupta *et al.* (2019) [Bhunya SGM, 55.98% and 66.95%]. The proposition of Min. $RE(Q_{PS})$ as an ob-

Table 2. Calibration & validation parameters of the SGMs and their statistical performance evaluation using observed and computed SG properties

Event Date	Model	Parameters							Total sediment outflow Q_s (kg/s)			Peak sediment outflow rate Q_{PS} (kg/s)/h			Time to Peak t_{PS} (h)			NSE (%)
		α	β	κ	θ	λ	S	A	Obs.	Comp.	RE (%)	Obs.	Comp.	RE (%)	Obs.	Comp.	RE (%)	
10-11/07/1981	SMA-SGM1	0.47	0.10	0.04				5.68	2182.58	2024.59	7.24	144.32	144.32	0.00	1	1	0.00	95.66
Calibration	SMA-SGM2	0.15	0.10	0.04	0.04		372.70	5.68	2182.58	2024.59	7.24	144.32	144.32	0.00	1	1	0.00	95.66
	SMA-SGM3	0.15	0.10	0.04		0.04	372.70	5.61	2182.58	2024.59	7.24	144.32	144.32	0.00	1	1	0.00	95.66
	SMA-SGM4	0.15	0.10	0.04	0.04	0.04	372.70	5.68	2182.58	2024.59	7.24	144.32	144.32	0.00	1	1	0.00	95.66
24-26/07/1982	SMA-SGM1	0.49	0.10	0.04				3.78	1452.76	1420.30	2.23	119.42	119.41	0.00	2	2	0.00	92.49
Calibration	SMA-SGM2	0.15	0.10	0.04	0.04		372.70	3.78	1452.76	1420.30	2.23	119.42	119.41	0.00	2	2	0.00	92.49
	SMA-SGM3	0.15	0.10	0.04		0.04	372.70	3.78	1452.76	1420.30	2.23	119.42	119.42	0.00	2	2	0.00	92.49
	SMA-SGM4	0.15	0.10	0.04	0.04	0.04	372.70	3.78	1452.76	1420.30	2.23	119.42	119.42	0.00	2	2	0.00	92.49
21-22/08/1983	SMA-SGM1	0.48	0.10	0.04				52.79	20295.35	20136.50	0.78	1317.46	1311.74	0.43	7	7	0.00	92.18
Validation	SMA-SGM2	0.15	0.10	0.04	0.04		372.70	52.79	20295.35	20136.49	0.78	1317.46	1311.74	0.43	7	7	0.00	92.18
	SMA-SGM3	0.15	0.10	0.04		0.04	372.70	52.79	20295.35	20136.50	0.78	1317.46	1311.74	0.43	7	7	0.00	92.18
	SMA-SGM4	**0.15**	**0.10**	**0.04**	**0.04**	**0.04**	**372.70**	**52.79**	**20295.35**	**20136.53**	**0.78**	**1317.46**	**1311.74**	**0.43**	**7**	**7**	**0.00**	**92.18**

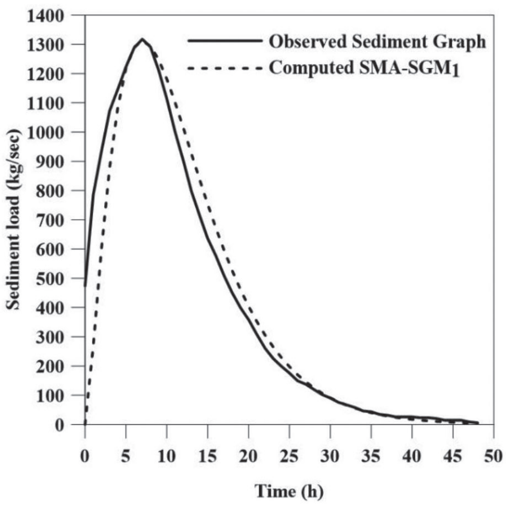

Fig. 3. Observed *vs.* Computed Sediment Graph of Validation Event (21-22/08/1983)

jective function rather than mimicking of SG using conventional *NSE*, underlines its significance in predicting the realistic SG besides conserving its mass reliably confirming t_p. Thus, proposed SMA-SGMs with altered objective function found more convenient in simulating sediment response of an arid basin.

Conclusion

1. The estimation of storm induced sediment response of the arid drainage basin using proposed SMA-SGMs is promising and exhibit their adequacy.
2. Hydrologically enhanced SMA-SGMs reproduces SGs peaks perfectly and timely by conserving the mass justifiably (or nearly) over a given time base as evidenced by negligible *REs* in Q_S, Q_{PS}, t_{PS} i.e. 7.24%, 0.43%, and 0, respectively.
3. When *S* is fixed using known CN, event specific *A* derived using SDR concept and the rest averaged calibration parameters α, β, k, θ and λ ($\approx$ 0.48 & 0.15, 0.10, 0.04, 0.04 and 0.04, respectively) are applicable to geomorphologically identical ungauged watersheds.
4. Thus, parameter driven model complexity is reduced by preserving their uniqueness/consistency adhering to their literature-cited values/ranges and enhancing their physical basis by skipping/constraining conventional direct optimization of *S* and *A*.
5. The proposed gamma distribution based smooth SMA-SGMs justifiably substitutes the observed sharp peaked oscillatory SGs. These results further strengthen suitability of the proposed SMA-SGMs in prediction of SG and total sediment outflow from ungauged arid basins.

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Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

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