Best compromised reservoir operating policy under uncertain inflows by Fuzzy Linear Programming

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ABSTRACT

The operation of the reservoir is a complex phenomenon, in which the operator should take suitable decisions to release water for various users to satisfy their objectives. This is further complicated if uncertainties are involved in the parameters of the reservoir operation process. In order to obtain the compromised optimal solution addressing the uncertainties involved in the parameters of the system, Fuzzy optimization approach is very useful. Compromised solution is obtained by Fuzzy Linear Programming (FLP) model considering inflows are in fuzzy. Firstly, the crisp solution is obtained by linear programming by solving the objective function considering for each set of uncertain inflows and corresponding best and worst values of objective are determined. Then objective function and reservoir inflows are fuzzified by considering linear membership function. Finally best compromised policy is found by maximizing the level of satisfaction (λ) considering the modified fuzzy objective and fuzzy inflows by FLP. The above model is applied to Nagarjuna Sagar reservoir on Krishna River located at border of Andhra Pradesh and Telangana state, India. The objective function considered in the present study is maximization of total irrigation releases subjected to irrigation releases, power releases, storage bounds and continuity constraints. The 75% and 80% dependable inflows are considered in the present study for determining the compromised reservoir operating policy. The maximum level of satisfaction (λ) obtained by simultaneously optimizing the fuzzified objective function and inflows is 0.5 and best compromised monthly operating policy is presented.

Key words: Reservoir operation, Linear programming, Fuzzy logic, Compromised policy, Uncertainty

Introduction

Fresh water availability is very less on the earth to meet domestic, irrigation, hydro-power, industrial and environmental flow requirements. This necessitates utilization of the limited available water in an optimal manner. Loucks *et al.* (1981) has given application of mathematical modeling methods in water management. Labadie (2004) has given a detailed review on state-of-the-art review for optimal multi-reservoir operation. The conventional optimization methods like Linear Programming, Dynamic Programming, Non- Linear programming and Meta-heuristic techniques will give crisp solution for the given objective function and constraints. These methods are not able to find the compromised solution when the objectives or variables are in fuzzy nature. Fuzzy Logic is found to be best tool for obtaining the compromised solution when the objectives and variables involved in the process are fuzzy in nature. Many researchers carried out research work on reservoir operation optimization in association with fuzzy logic approach. The concept of fuzzy decision-making was first proposed by Bellman and Zadeh (1970). Fuzzy Linear Programming (FLP) is first introduced by Zimmermann (1978). Rommelfanger (1996) has addressed detailed survey on fuzzy linear system techniques. Srinivasa Raju and Nagesh Kumar (2000) developed a FLP model considering vagueness and imprecision on objective function values. Nagesh Kumar et al. (2001) have developed Multi-Objective Fuzzy Linear Programming (MOFLP) model in which only objectives are considered as fuzzy and constraints are in crisp nature. Mohan and Jothiprakash (2000) have developed FLP model for optimal cropping pattern where inflows and ground water flows are considered as fuzzy. Kamodkar and Regulwar (2010) have developed three models by FLP in reservoir operation considering fuzziness in objectives, resources and technological coefficients respectively and compared the level of satisfaction (λ) obtained with the three models.

In the present study, best compromised reservoir operating policy is obtained by FLP by considering inflows entering into the reservoir are in fuzzy. The objective of the study is maximization of annual releases for irrigation. The model is optimized for maximum level of satisfaction by considering fuzzified objective and inflows are constraints. The above developed model is applied and demonstrated through a case study of Nagarjuna Sagar Reservoir on River Krishna to obtain best compromised reservoir operating policy.

Materials and Methods

Study Area

The physical system considered in the present study is Nagarjuna Sagar Reservoir built across River Krishna located at border of Guntur district in Andhra Pradesh and Nalgonda district in Telangana states, India, as shown in Figure 1. The longitude and lattitude of the reservoir are 16°34'32"N and 79°18'42" E. The dam is 180 m height from its foundation and 1.6 km long with 26 flood gates which are of size 13mx14m. The gross storage of the reservoir is 11560 Mm³ (MCM) out of this the live storage is 5730 MCM. The reservoir catchment area is 215000 km² and water surface area at full reservoir level is 285 km². The full reservoir level and minimum draw down level are 179.3 m and 156.3 m, respectively. Two main irrigation canals take off from the reservoir on either side canal (left canal) both having equal carrying capacity of 311.5 m³/sec. The length of left main canal is 179 km and irrigates 0.3869 million hectares (M Ha) while it is 203 km and irrigates 0.4505 M Ha for right main canal. The major crops grown under the canal command area in different seasons are Rice, Cotton, Chilli, Groundnut, Sorghum and Grams. The main power plant is located on river downstream side (d/s) with a capacity of 810 MW having 8 units (1x110MW+7x100 MW). Two minor power plants of capacity 60 MW and 90 MW are located on left and right main canals respectively. The inflows of 75% and 80% reliability, irrigation demands and minimum d/s release requirements are computed and are presented in Table 1.



Fig. 1. Layout of Nagarjuna Sagar Reservoir project

Model Formulation

Monthly steady state optimal reservoir operating policy is formulated considering a planning horizon of a year. The model is formulated to find compromised monthly optimal releases into irrigation canals maximizing the annual releases for irrigation considering uncertain inflows.

Objective function, (Z) = maximization of totalsum of irrigation releases = $\sum_{t=1}^{12} IR_t$ (1)where, IR, are irrigation releases in a time period t.

Subject to the following constraints,

Lower and upper bounds for irrigation release $O.5*ID_{+} \leq IR_{+} \leq ID_{+}$

where, *ID*, is the irrigation demand in any time

Month	Reservoir Inflows		Irrigation demand	Minimum d/s release
	75% PE	80% PE	ID(t)	DRMIN(t)
JUL	1175.00	652.15	566.27	178.74
AUG	3201.52	2860.70	1603.27	178.74
SEP	2458.60	2364.50	1592.47	178.74
OCT	2255.04	2109.86	1747.74	178.74
NOV	888.36	758.80	1980.50	178.74
DEC	615.00	452.70	1444.14	178.74
JAN	394.60	348.67	268.35	178.74
FEB	292.16	267.44	224.50	178.74
MAR	217.00	201.80	65.79	178.74
APR	174.10	158.40	0.0	178.74
MAY	88.10	75.37	0.0	178.74
JUN	156.84	150.87	0.0	178.74
Total	11916.32	10401.26	9493.04	2144.92

Table 1. Dependable Inflows, Irrigation demands & min. d/s releases required (MCM)

period t and the minimum irrigation release considered in the study is 50% of its demand.

Lower and upper bounds for power release

$$DRMIN_{t} \le DR_{t} \le DRMAX \qquad .. (3)$$

where, $DRMIN_t$ and DRMAX are minimum and maximum d/s releases required in any time period t respectively.

Active storage constraint

$$0 \le S_{t} \le SMAX \qquad \dots (4)$$

where S_t is the active storage in any time period t and should not be more than active storage capacity (*SMAX*) of the reservoir.

Storage continuity equation

$$(1+a_t)St_{+1} = (1-a_t)S_t + Q_t - IR_t - DR_t - SP_t - e_t$$
 ... (5)

where, Q_t , SP_t and e_t are inflow, spill and evaporation in a time period t respectively.

Steady state storage condition

$$S_{T+1} = S_1$$
 ... (6)

i.e. The storage at the end of the present year is equal to the storage at the beginning of the next year.

Fuzzy Linear Programming

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The standard Linear Programming problem is,

$$Max. Z = \sum_{j=1}^{n} c_j x_j \qquad \dots (7)$$

Subject to
$$\sum_{i=1}^{n} a_{i,j} x_j d'' b_i$$
, ... (8)

$$x_i \ge 0. \qquad \qquad \dots (9)$$

By solving the above model a crisp solution is

obtained for the given conditions. But in real case, the objective function values, resources (right hand side of the constraint equations i.e. b) and technological coefficients (left hand side coefficient matrix of constraint equations i.e. a_i) may be uncertain, imprecise or fuzzy in nature. Prof. L.A. Zadeh introduced the concept of Fuzzy sets in 1965. The principle of the Fuzzy logic is, it allows something to be partly this and partly that, rather than having to be either all this or all that. It is the degree of "belongingness or membership" to a set expressed by a real number in between the interval 0 to 1. Usually the membership function is assumed to be linear, making computations simpler. FLP is the most appropriate method to incorporate fuzziness in the model parameters assuming a linear membership function for obtaining the best compromised solution. In the present case, best compromised reservoir operating policy is obtained by FLP by considering inflows entering into the reservoir are in fuzzy. The step by step procedure of FLP model is as follows,

Step 1:

$$Max. Z = \sum_{j=1}^{n} c_j x_j \qquad \dots (10)$$

Subject to
$$\sum_{i=1}^{m} a_{i,j} x_j d'' \widetilde{b_{j'}}$$
 .. (11)

$$x_i \ge 0$$
 ... (12)

In this case, is a fuzzy numbers with linear membership function shown in Figure 2.

Step 2

Best and worst values of the objective function are



Fig. 2. Linear membership function for resources (b_i)

to be determined corresponding to the lower and upper limits imposed on resources as per the standard procedure of LP. The graphical representation of membership function for the objective function values is shown in Figure 3.



Fig. 3. Linear membership function for objective function values

Step 3

The LP problem is re-formulated as, Max. l (Level of satisfaction)

s.to.
$$\mu Z(x) \le \lambda$$
 .. (13)

$$\mu C_i(x) \le \lambda \qquad \qquad \dots (14)$$

$$0 \le \lambda \le \lambda \qquad \qquad \dots (15)$$

in addition to all other original constraints.

Step 4

By solving the above re-formulated LP problem, the best compromised solution corresponding to the maximum level of satisfaction (λ) is obtained.

Results and Discussion

The above discussed FLP model is applied to

Nagarjuna Sagar Reservoir considering maximizing the annual irrigation releases. The best compromised reservoir operating policy considering fuzziness in the objective and reservoir inflows is

By following above procedure of FLP, The model is solved by LP taking each set of inflows at a time and the corresponding best and worst values of objective function are obtained. The best and worst values of the objective are 8827.318 MCM, 7304.08 MCM. After obtaining the bounds for the objectives, the objectives and inflows are fuzzified by assuming linear membership function. The membership functions for objective function & inflows are shown in Figures 4 & 5.

obtained by MATLAB software.



$$\mu_{F_{1}}(x) = \begin{cases} 0 & F_{1} \leq 7304.08\\ \left(\frac{F_{1}-7204.08}{8827.318-7304.08}\right) & 7304.08 < F_{1} < 8827.318\\ 1 & F_{1} \geq 8827.318\\ \dots & (16) \end{cases}$$

Membership function value for inflows is considered as '1' for all flows having reliability greater than or equal to 80% and '0' for flows less than 75% reliability as presented in Figure 5.



Fig. 5. Membership function for inflows

The LP model is re-formulated to find the maximum level of satisfaction (λ) by using membership function equations (16) & (17).

λ

Maximize

Subject to F-
$$1523.238\lambda - 7304.08 \le 0$$
 (18)

$$Q_{80\%i} - (Q_{75\%i} - Q_{80\%i}) \lambda - Q_i \le 0 \forall i, i = 1 \text{ to } 12 \dots (19)$$

$$0 \le \lambda \le 1 \qquad \qquad \dots (20)$$

In addition to the above three constraints, the equations (1) to (6) in the earlier model are also considered. The modified LP problem is solved for getting the best compromised solution. The level of satisfaction (λ) obtained is 0.5 and the corresponding objective function value is 8066.49 MCM respectively. The best compromised reservoir operating policy is shown in Table 2. It shows that the total irrigation releases by FLP model are 8.62% lower when compared to LP model with 75% reliable inflows while they are 10.43% higher when compared to LP model with 80% reliable inflows.

 Table 2.
 Optimal release policies by LP & FLP Models in MCM

Month]	LP	FLP
	75% PE	80% PE	policy
JUL	369.501	283.138	283.138
AUG	1603.270	1603.270	1603.27
SEP	1592.470	1592.470	1592.47
OCT	1747.740	1747.740	1747.74
NOV	1980.500	1076.067	1838.476
DEC	1254.512	722.070	722.07
JAN	134.177	134.177	134.177
FEB	112.250	112.250	112.25
MAR	32.898	32.898	32.898
APR	0.000	0.000	0
MAY	0.000	0.000	0
JUN	0.000	0.000	0
Total	8827.318	7304.080	8066.489

The monthly active storages to be maintained in the reservoir by LP (75% & 80%) and FLP are presented in Figure 6. It shows that the reservoir storage is increasing from AUG to NOV because of the monsoon inflows into the reservoir and thereafter it is getting depleted in all cases. The storages required to be maintained are higher in FLP with respect to 80% reliable inflows and lower with 75% reliable inflows from AUG to NOV and thereafter follows reverse nature.



Fig. 6. Monthly Active storages in the reservoir

Conclusion

Fuzzy Linear Programming model is formulated and applied to Nagarjuna Sagar reservoir to find best compromised reservoir operating Policy under uncertain inflows. The objective function considered is maximization of total irrigation releases. The maximum level of satisfaction is obtained as 0.5. The monthly active storages to be maintained in the reservoir and best compromised releases for irrigation are determined. This policy will be useful to the decision makers as the inflows are in fuzzy.

References

- Bellman, R.E. and Zadeh, L.A. 1970. Decision Making in a Fuzzy Environment. *Management Sciences*. 17: 141-164.
- Loucks, D. P., Stedinger, J. and Haith, D.A. 1981. Water Resources Systems Planning and Analysis. *Prentice-Hall, Eaglewood Cliffs*, NJ.
- Zimmermann, H. J. 1996. Fuzzy Set Theory and Its Applications. Allied Publishers, New Delhi.
- Rommelfanger, H. 1996. Fuzzy Linear Programming and Applications. European Journal of Operational Research. 92 (3): 512-527.
- Labadie, J. W. 2004. Optimal Operation of Multi-reservoir System: State of the Art Review. *Journal of Water Resource Planning and Management*. 130 (2): 93-111.
- Srinivasa Raju, K. and Nagesh Kumar, D. 2000. Irrigation Planning of Sri Ram Sagar Project Using Multi-objective Fuzzy Linear Programming. *Journal of Hydraulic Engineering*. 6 (1): 55-62.

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Kamodkar and Regulwar. 2010. Derivation of Multipurpose Single Reservoir Release Policies with Fuzzy Constraints. J. Water Resource and Protection. 2:1030-1041.

Mohan, S. and Jothiprakash, V. 2000. Fuzzy System Mod-

eling for Optimal Crop Planning. *Journal of Institution of Engineers (India)*. 81(3) : 9-17.

Yeh, W. 1985. Reservoir management and operations models: A state-of-the-art review, *Water Resources Research J.* 21(12) : 1797-1818.